



# Derivative time-domain sensor and fibre optic correction factor calculation

Marc Sallin and Bertrand Daout, montena technology, 1728 Rossens, Switzerland

### Table of contents

1. Introduction .....	1
2. Measurement setup .....	2
3. Ground-field D-dot sensor.....	3
4. Ground-field B-dot sensor.....	3
5. Free field D-dot sensor .....	4
6. Free field B-dot sensor.....	5
7. Balun.....	6
8. Attenuators and coaxial cables .....	6
9. Optical fibre, transmitter and receiver .....	7
10. Oscilloscope.....	8
11. Correction factor (free field D-dot sensor).....	9
12. Correction factor (free field B-dot sensor) .....	10
13. Correction factor (ground field D-dot sensor).....	11
14. Correction factor (ground field B-dot sensor) .....	11
15. Attenuator value selection (free field D-dot sensor) .....	12
16. Attenuator value selection (free field B-dot sensor) .....	13
17. Attenuator value selection (ground field D-dot sensor) .....	14
18. Attenuator value selection (ground field B-dot sensor) .....	14

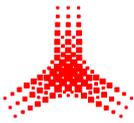
## 1. Introduction

The measurement of pulsed electric or magnetic field is a delicate task that requires some signal post-processing. In order to assess the measured field from the value read on the oscilloscope, several correction factors must be taken into account in the calculation. This technical note explains the source of the correction factors and how to use them in an appropriate way.

This technical note reviews the following types of sensors:

- Ground field electric time-domain sensors
- Ground field magnetic time-domain sensors
- Free field electric time-domain sensors
- Ground field electric time-domain sensors

This technical note refers to electric and magnetic time-domain sensors which are not calibrated with respect to a specific field, i.e. their mechanical dimensions is sufficient for a complete determination of their behaviour. The main parameter of this type of sensor is called the “equivalent surface” and is only linked to the mechanical dimension of the receiving elements.

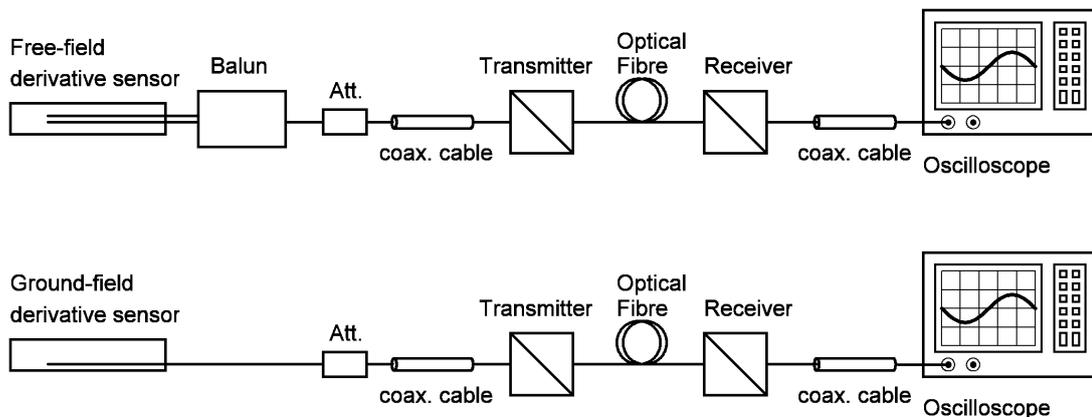


For explanation about sensors which are calibrated with respect to a specific field and that are delivered with a calibration factor, please refer to TN21 “Derivative time-domain sensor calibration procedure” for further details.

Please note that all the sensors can either be used with their equivalent surface or with their calibration factor.

## 2. Measurement setup

The general schematic of the measurement chain for a derivative sensor is shown below. The same principle can be applied for a D-dot and a B-dot sensor (ground field or free field sensor). Please note however that the balun is not needed in the case of the ground field sensors.



It consists in the following devices:

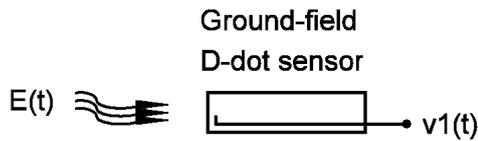
- a D-dot or B-dot (ground field or free field) sensor
- a balun (only needed in the case of free field sensors)
- an attenuator
- several coaxial cables
- an optical transmitter
- a fibre optic
- an optical receiver
- an oscilloscope (with integration function capabilities)

Each above mentioned device has a correction factor that must be used in the calculation of the field. After a quick review of the four types of sensor and their related equations, the correction factor for each device is detailed in the following paragraphs.

The role of the attenuator located between the sensor and the fibre optic is the protection of the transmitter input, which is limited by manufacturer specification. The choice of the minimum attenuator value is also derived below.



### 3. Ground-field D-dot sensor



The ground field D-dot sensor transforms an electric field into a voltage, whose shape is the derivative of the electric field waveform. This type of sensor must be located over a ground plane and consists in only one single channel. Its equation is the following:

$$v_1(t) = R_s \cdot A_{eq,s} \frac{\partial D(t)}{\partial t} \quad \text{with } D(t) = \epsilon_0 \cdot E(t)$$

where:

$R_s$  is the impedance seen by a single channel of the sensor in  $\Omega$ .

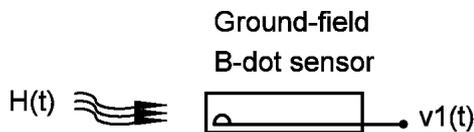
$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

$D$  is the electric displacement.

$\epsilon_0$  is the electric permittivity of free space.

More details about the derivation of that formula can be found in TN12 “Electric and magnetic field sensor and integrator equations”.

### 4. Ground-field B-dot sensor



The ground field B-dot sensor transforms a magnetic field into a voltage, whose shape is the derivative of the magnetic field waveform. This type of sensor must be located over a ground plane and consists in only one single channel. Its equation is the following:

$$v_1(t) = A_{eq,s} \frac{\partial B(t)}{\partial t} \quad \text{with } B(t) = \mu_0 \cdot H(t)$$

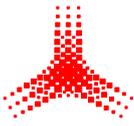
where:

$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

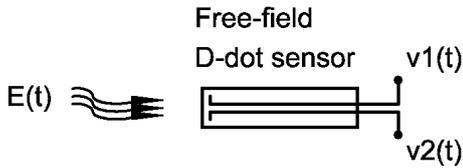
$B$  is the magnetic flux density.

$\mu_0$  is the magnetic permeability of free space.

More details about the derivation of that formula can be found in TN12 “Electric and magnetic field sensor and integrator equations”



## 5. Free field D-dot sensor



The free field D-dot sensor transforms an electric field into a voltage, whose shape is the derivative of the electric field waveform.

A free field sensor is composed of two identical channels, which are sensitive to the same field, but measure in opposite directions. The equations for both channels are the following:

$$v_1(t) = R_s \cdot A_{eq,s} \frac{\partial D(t)}{\partial t} \quad v_2(t) = -R_s \cdot A_{eq,s} \frac{\partial D(t)}{\partial t} \quad \text{with } D(t) = \varepsilon_o \cdot E(t)$$

where:

$R_s$  is the impedance seen by a single channel of the sensor in  $\Omega$ .

$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

$D$  is the electric displacement.

$\varepsilon_o$  is the electric permittivity of free space.

The output voltage across both channels is the following:

$$v_1(t) - v_2(t) = 2 \cdot R_s \cdot A_{eq,s} \cdot \varepsilon_o \cdot \frac{\partial E(t)}{\partial t}$$

### **Remark:**

**Free field sensors are either specified with the equivalent area of a single sensor  $A_{eq,s}$  or with the total equivalent area of the sensor  $A_{eq,tot}$ . It is important to verify which value must be taken into account for the calculation.**

The relation between the equivalent area of a single channel and the total equivalent area is the following:

$$A_{eq,tot} = 2 \cdot A_{eq,s}$$

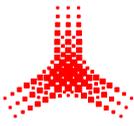
where:

$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

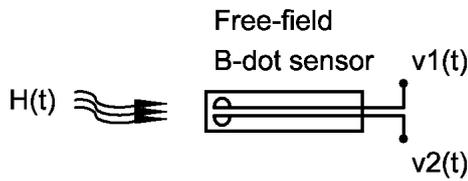
$A_{eq,tot}$  is the total equivalent area of the sensor in  $m^2$ , also called "differential equivalent area".

In case the sensor is specified with the total equivalent area, the sensor equation turns to:

$$v_1(t) - v_2(t) = R_s \cdot A_{eq,tot} \cdot \varepsilon_o \cdot \frac{\partial E(t)}{\partial t}$$



## 6. Free field B-dot sensor



The free field B-dot sensor transforms a magnetic field into a voltage, whose shape is the derivative of the magnetic field waveform.

A free field sensor is composed of two identical channels, which are sensitive to the same field, but measure in opposite directions. So the equations for both channels are the following:

$$v_1(t) = A_{eq,s} \frac{\partial B(t)}{\partial t} \quad v_2(t) = -A_{eq,s} \frac{\partial B(t)}{\partial t} \quad \text{with } B(t) = \mu_o \cdot H(t)$$

where:

$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

$B$  is the magnetic flux density.

$\mu_o$  is the magnetic permeability of free space.

The output voltage across both channels is the following:

$$v_1(t) - v_2(t) = 2 \cdot A_{eq,s} \cdot \mu_o \cdot \frac{\partial H(t)}{\partial t}$$

**Remark:**

**Free field sensors are either specified with the equivalent area of a single sensor  $A_{eq,s}$  or with the total equivalent area of the sensor  $A_{eq,tot}$ . It is important to verify which value must be taken into account for the calculation.**

The relation between the equivalent area of a single channel and the total equivalent area is the following:

$$A_{eq,tot} = 2 \cdot A_{eq,s}$$

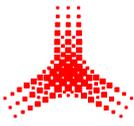
where:

$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

$A_{eq,tot}$  is the total equivalent area of the sensor in  $m^2$ , also called "differential equivalent area".

In case the sensor is specified with the total equivalent area, the sensor equation turns to:

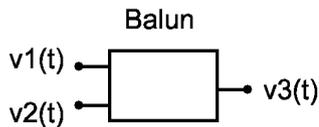
$$v_1(t) - v_2(t) = A_{eq,tot} \cdot \mu_o \cdot \frac{\partial H(t)}{\partial t}$$



## 7. Balun

The balun is needed to transform two balanced signals  $v_1(t)$  and  $v_2(t)$  into a single unbalanced signal. It is only needed when free field sensors are used. As the ground field sensors have only one single channel  $v_1(t)$ , they can directly be connected to rest of the measurement circuit.

Because of the internal topology and the input filtering of this circuit, this transformation reduces the signal level.



This device can therefore be summarized with the following equation:

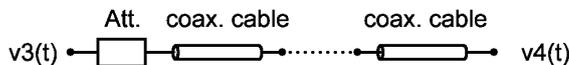
$$v_3(t) = \frac{v_1(t) - v_2(t)}{10^{\frac{K_{Bal}}{20}}}$$

where:

$K_{Bal}$  is the attenuation of the balun in dB.

## 8. Attenuators and coaxial cables

The value of the attenuator must be chosen so that the voltage  $v_4(t)$  is always below the maximum input voltage of the transmitter. (see paragraph 15 for more details). If not, the voltage signal might be distorted and therefore give wrong results.



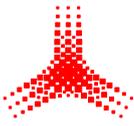
$$v_4(t) = \frac{v_3(t)}{10^{\frac{K_{Att}}{20}}}$$

where:

$K_{Att}$  is the attenuation of the attenuator in dB.

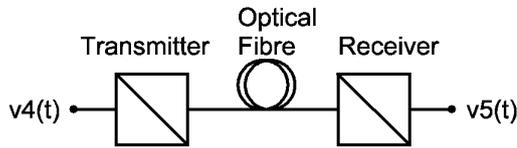
The losses of the attenuator and the coaxial cables forming the measurement chain can be considered as a single attenuation. At high frequencies, coaxial cables can no more be seen as perfect elements, as they show some attenuation which is dependant on the frequency (low-pass effect). Therefore all cables must be taken into account for the calculation of the correction factor.

$K_{Att}$  must be measured with a network analyser, in order to check the deviation of the attenuation from the nominal value.



## 9. Optical fibre, transmitter and receiver

The fibre optic is used because the metallic coaxial cable disturbs the electromagnetic wave to be measured or if the distance between the sensor and the oscilloscope is too large. It consists of a transmitter, a fibre optic and an optical receiver.



The equation is the following:

$$v_5(t) = \frac{v_4(t)}{10^{\frac{K_{Opt}}{20}}}$$

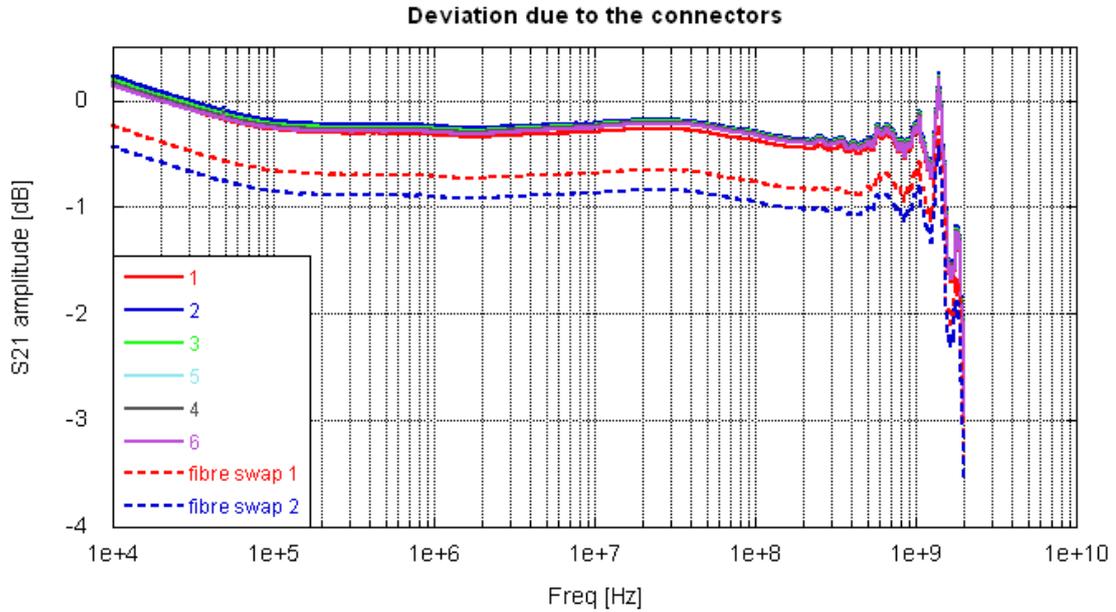
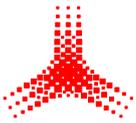
where:

$K_{Opt}$  is the attenuation of the whole optical link in dB.

$K_{Opt}$  must be measured with a network analyser. Special care must be taken when measuring the fibre optic link:

- Allow at least 20 minutes warm-up after the transmitter and receiver are switched on.
- The output signal of the network analyser must be below the maximum input voltage allowed by the transmitter.
- Due to their internal electronic circuit, some fibre optic links can even show some gain for certain frequencies.
- For some fibre optic links,  $K_{Opt}$  is extremely sensitive to any variation of the optical connectors. The specified variation of attenuation for different connections is  $\pm 1$  dB. If precise measurements are needed, it is recommended not to disconnect the optical connectors, once the attenuation measurement is performed. Some advanced fibre optic links can perform a self-calibration. In that case, the variation due to the connectors is included in the calibration and the problem disappears. Some fibre optic links have an automatic gain control feature that compensates for the connection losses. This is the case for the montena fibre optic links like the MOL3000 and MOL3000-26. Also in that case, the problem disappears.

To illustrate this last point, a series of measurement was performed with a fibre optic link that does not have a self-calibration nor automatic gain control feature. The following graph shows the S21 vs frequency for the fibre optic link for 8 different optical connections. Between each measurement, the optical connectors are only disconnected and reconnected. In this case, the deviation is about less than 1 dB.

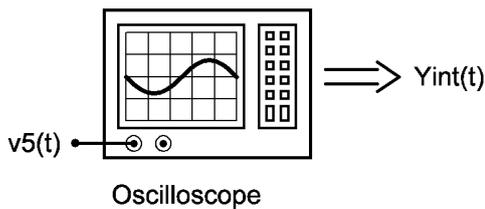


A solution to improve the repeatability is to always connect the fibre optic in the same direction, i.e. one end of the cable always to the transmitter and the other end always to the receiver.

As visible from the above graph, the response of the fibre optic link is not flat over the specified frequency range. The attenuation can be approximated by an average of the attenuation over the frequency range of the measurement signal.

## 10. Oscilloscope

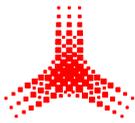
The oscilloscope displays the voltage  $v_5(t)$ , which is in fact an image of the derivative of the measured electric field. In order to go back to the electric field pulse shape, the signal must be integrated with a special function of the oscilloscope. This results in  $Y_{int}(t)$ , whose unit is [Vs] (Volt x second).



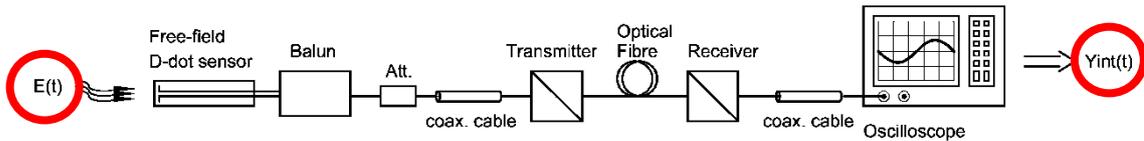
$$Y_{int}(t) = \int (v_5(t) + k) \cdot dt$$

The channel coupling must be set to 50  $\Omega$ .

An adequate integration constant  $k$  must be given to the integration function of the oscilloscope.



## 11. Correction factor (free field D-dot sensor)



According to all the above mentioned contributions, the electric field waveform can be recovered from the integrated oscilloscope measurement with the following formula:

$$E(t) = \frac{10^{\frac{K_{Bal} + K_{Att} + K_{Opt}}{20}}}{2 \cdot R_s \cdot A_{eq,s} \cdot \epsilon_0} \cdot Y_{int}(t)$$

where:

$Y_{int}(t)$  is the integrated voltage at the oscilloscope display in Vs.

$R_s$  is the impedance seen by a single channel of the sensor in  $\Omega$ .

$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

$K_{Bal}$  is the attenuation of the balun in dB.

$K_{Att}$  is the attenuation of the combination of the attenuator and the coaxial cables in dB.

$K_{Opt}$  is the attenuation of the fibre optic link in dB.

$\epsilon_0$  is the permittivity of free space ( $8.85 \times 10^{-12}$  F/m)

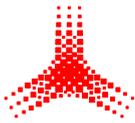
Example

- Impedance seen by a single channel of the sensor: 50  $\Omega$
- Equivalent area of a single channel of the sensor:  $10^{-3}$   $m^2$
- Balun attenuation: 8 dB
- Attenuator: 40 dB
- Optical link attenuation: 1 dB

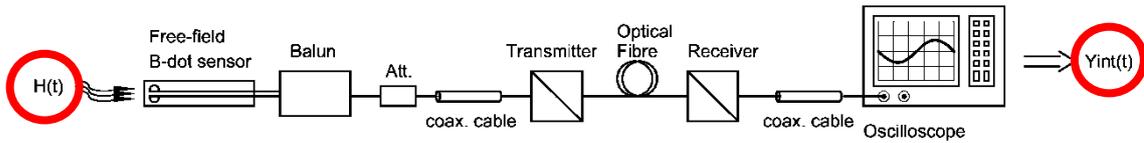
$$E(t) = \frac{10^{\frac{8dB + 40dB + 1dB}{20}}}{2 \cdot 50 \cdot 10^{-3} \cdot 8.85 \cdot 10^{-12}} \cdot Y_{int}(t)$$

$$E(t) = \underline{3.18 \cdot 10^{14}} \cdot Y_{int}(t)$$

The correction factor is  $3.18 \cdot 10^{14} m^{-1}s^{-1}$ .



## 12. Correction factor (free field B-dot sensor)



According to all the above mentioned contributions, the magnetic field waveform can be recovered from the integrated oscilloscope measurement with the following formula:

$$H(t) = \frac{10^{\frac{K_{Bal} + K_{Att} + K_{Opt}}{20}}}{A_{eq,tot} \cdot \mu_0} \cdot Y_{int}(t)$$

where:

$Y_{int}(t)$  is the integrated voltage at the oscilloscope display in Vs.

$A_{eq,tot}$  is the total equivalent area of the sensor in  $m^2$ .

$K_{Bal}$  is the attenuation of the balun in dB.

$K_{Att}$  is the attenuation of the combination of the attenuator and the coaxial cables in dB.

$K_{Opt}$  is the attenuation of the fibre optic link in dB.

$\mu_0$  is the permeability of free space ( $1.256 \times 10^{-6}$  H/m)

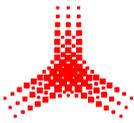
Example

- Total equivalent area of the sensor:  $9 \times 10^{-6} m^2$
- Balun attenuation: 8 dB
- Attenuator: 40 dB
- Optical link attenuation: 1 dB

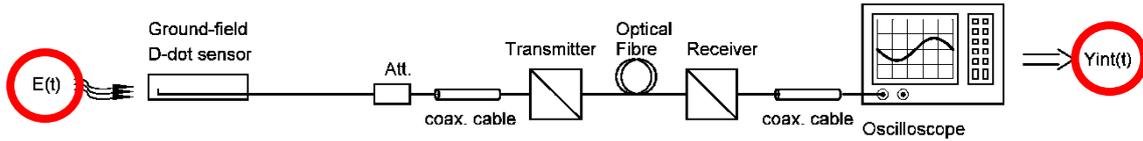
$$H(t) = \frac{10^{\frac{8dB + 40dB + 1dB}{20}}}{9 \cdot 10^{-6} \cdot 1.256 \cdot 10^{-6}} \cdot Y_{int}(t)$$

$$H(t) = \underline{2.49 \cdot 10^{13}} \cdot Y_{int}(t)$$

The correction factor is  $\underline{2.49 \cdot 10^{13} \Omega^{-1} m^{-1} s^{-1}}$ .



### 13. Correction factor (ground field D-dot sensor)



Using the same as in Paragraph 11, but setting the balun attenuation to zero and taking into account only one single channel leads to the following equation:

$$E(t) = \frac{10^{\frac{K_{Att} + K_{Opt}}{20}}}{R_s \cdot A_{eq,s} \cdot \epsilon_0} \cdot Y_{int}(t)$$

$Y_{int}(t)$  is the integrated voltage at the oscilloscope display in Vs.

$R_s$  is the impedance seen by a single channel of the sensor in  $\Omega$ .

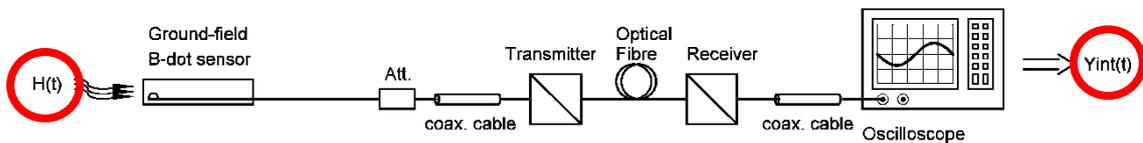
$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

$K_{Att}$  is the attenuation of the combination of the attenuator and the coaxial cables in dB.

$K_{Opt}$  is the attenuation of the fibre optic link in dB.

$\epsilon_0$  is the permittivity of free space ( $8.85 \times 10^{-12}$  F/m)

### 14. Correction factor (ground field B-dot sensor)



Using the same as in Paragraph 12, but setting the balun attenuation to zero and taking into account only one single channel leads to the following equation:

$$H(t) = \frac{10^{\frac{K_{Att} + K_{Opt}}{20}}}{A_{eq,s} \cdot \mu_0} \cdot Y_{int}(t)$$

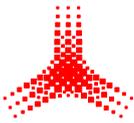
$Y_{int}(t)$  is the integrated voltage at the oscilloscope display in Vs.

$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

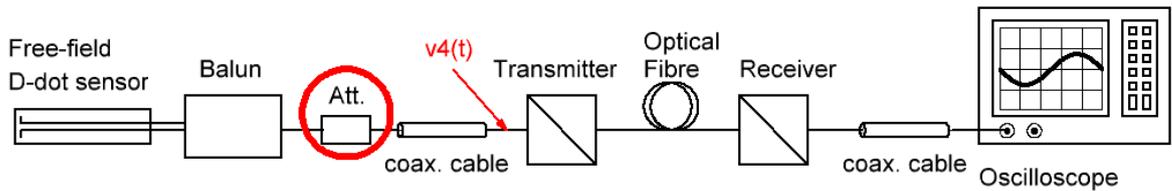
$K_{Att}$  is the attenuation of the combination of the attenuator and the coaxial cables in dB.

$K_{Opt}$  is the attenuation of the fibre optic link in dB.

$\mu_0$  is the permeability of free space ( $1.256 \times 10^{-6}$  H/m)



## 15. Attenuator value selection (free field D-dot sensor)



The attenuator must be chosen so that the input voltage of the fibre optic transmitter  $v_4(t)$  is always below the maximum voltage  $V_{\max, \text{transmitter}}$ :

$$v_4(t) < V_{\max, \text{transmitter}}$$

Expanding the equation for  $v_4(t)$ :

$$\frac{2 \cdot R_s \cdot A_{\text{eq},s} \cdot \epsilon_0 \cdot \frac{\partial E(t)}{\partial t}}{10^{\frac{K_{\text{Bal}} + K_{\text{Att}}}{20}}} < V_{\max, \text{transmitter}}$$

The time-derivative of the electric field can be approximated by the following expression:

$$\frac{\partial E(t)}{\partial t} \cong \frac{\Delta E(t)}{\Delta t} \cong \frac{E_{\text{peak}}}{t_{\text{rise}}}$$

where:

$E_{\text{peak}}$  is the peak electric field value of the measured pulse in V/m.

$t_{\text{rise}}$  is the rise-time 10 % - 90 % of the measured pulse in s.

Finally the requested attenuation  $K_{\text{Att}}$  in dB can be derived as:

$$K_{\text{Att}} > 20 \cdot \log \left( \frac{2 \cdot R_s \cdot A_{\text{eq},s} \cdot \epsilon_0 \cdot E_{\text{peak}}}{V_{\max, \text{transmitter}} \cdot t_{\text{rise}}} \right) - K_{\text{Bal}}$$

where:

$R_s$  is the impedance seen by a single channel of the sensor in  $\Omega$ .

$A_{\text{eq},s}$  is the equivalent area of a single channel of the sensor in  $\text{m}^2$ .

$K_{\text{Bal}}$  is the attenuation of the balun in dB.

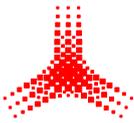
$V_{\max, \text{transmitter}}$  is the maximum allowed voltage at the input of the transmitter in V.

$\epsilon_0$  is the permittivity of free space ( $8.85 \times 10^{-12}$  F/m)

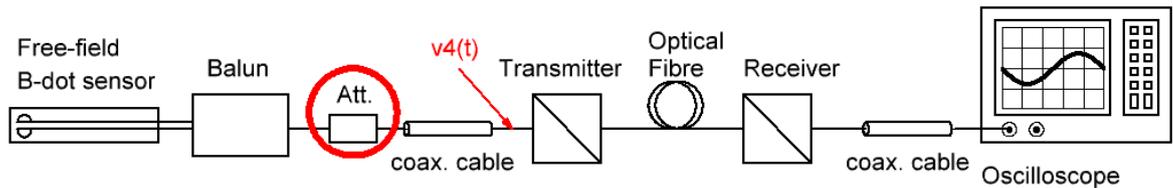
Example:

- Pulse peak electric field: 50 kV/m
- Pulse rise-time 10%-90%: 2 ns
- Impedance seen by a single channel of the sensor: 50  $\Omega$
- Equivalent area of a single channel of the sensor:  $10^{-3}$   $\text{m}^2$
- Balun attenuation: 8 dB
- Transmitter input peak voltage: 250 mV

$$K_{\text{Att}} > 20 \cdot \log \left( \frac{2 \cdot 50 \cdot 10^{-3} \cdot 8.85 \cdot 10^{-12} \cdot 50k}{250m \cdot 2n} \right) - 8dB = \underline{30.9 \text{ dB}}$$



## 16. Attenuator value selection (free field B-dot sensor)



The attenuator must be chosen so that the input voltage of the fibre optic transmitter  $v_4(t)$  is always below the maximum voltage  $V_{\max, \text{transmitter}}$ :

$$v_4(t) < V_{\max, \text{transmitter}}$$

Expanding the equation for  $v_4(t)$ :

$$\frac{A_{\text{eq,tot}} \cdot \mu_o \cdot \frac{\partial H(t)}{\partial t}}{10^{\frac{K_{\text{Bal}} + K_{\text{Att}}}{20}}} < V_{\max, \text{transmitter}}$$

The time-derivative of the magnetic field can be approximated by the following expression:

$$\frac{\partial H(t)}{\partial t} \cong \frac{\Delta H(t)}{\Delta t} \cong \frac{H_{\text{peak}}}{t_{\text{rise}}}$$

where:

$H_{\text{peak}}$  is the peak magnetic field value of the measured pulse in A/m.

$t_{\text{rise}}$  is the rise-time 10 % - 90 % of the measured pulse in s.

Finally the requested attenuation  $K_{\text{Att}}$  in dB can be derived as:

$$K_{\text{Att}} > 20 \cdot \log \left( \frac{A_{\text{eq,tot}} \cdot \mu_o \cdot H_{\text{peak}}}{V_{\max, \text{transmitter}} \cdot t_{\text{rise}}} \right) - K_{\text{Bal}}$$

where:

$A_{\text{eq,tot}}$  is the total equivalent area of the sensor in  $\text{m}^2$ .

$K_{\text{Bal}}$  is the attenuation of the balun in dB.

$K_{\text{Opt}}$  is the attenuation of the fibre optic link in dB.

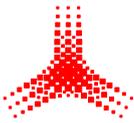
$V_{\max, \text{transmitter}}$  is the maximum allowed voltage at the input of the transmitter in V.

$\mu_o$  is the permeability of free space ( $1.256 \times 10^{-6}$  H/m).

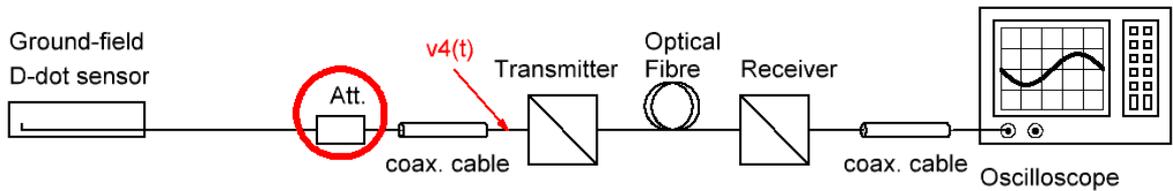
Example:

- Pulse peak magnetic field: 100 A/m
- Pulse rise-time 10%-90%: 100 ps
- Total equivalent area of the sensor:  $9 \times 10^{-6} \text{ m}^2$
- Balun attenuation: 8 dB
- Transmitter input peak voltage: 250 mV

$$K_{\text{Att}} > 20 \cdot \log \left( \frac{9 \cdot 100^{-6} \cdot 1.256 \cdot 10^{-6} \cdot 100}{250 \text{m} \cdot 100 \text{p}} \right) - 8 \text{dB} = \underline{25.1 \text{ dB}}$$



## 17. Attenuator value selection (ground field D-dot sensor)



Using the derivation of Paragraph 15 in the special case of the ground field D-dot sensor enables to compute the requested minimum attenuation value as:

$$K_{Att} > 20 \cdot \log \left( \frac{R_s \cdot A_{eq,s} \cdot \epsilon_0 \cdot E_{peak}}{V_{max, transmitter} \cdot t_{rise}} \right)$$

where:

$E_{peak}$  is the peak electric field value of the measured pulse in V/m.

$t_{rise}$  is the rise-time 10%-90% of the measured pulse in s.

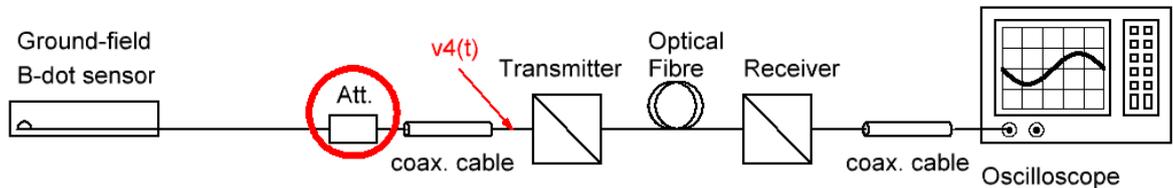
$R_s$  is the impedance seen by a single channel of the sensor in  $\Omega$ .

$A_{eq,s}$  is the equivalent area of a single channel of the sensor in  $m^2$ .

$V_{max, transmitter}$  is the maximum allowed voltage at the input of the transmitter in V.

$\epsilon_0$  is the permittivity of free space ( $8.85 \times 10^{-12}$  F/m)

## 18. Attenuator value selection (ground field B-dot sensor)



Using the derivation of Paragraph 16 in the special case of the ground field B-dot sensor enables to compute the requested minimum attenuation value as:

$$K_{Att} > 20 \cdot \log \left( \frac{A_{eq,s} \cdot \mu_0 \cdot H_{peak}}{V_{max, transmitter} \cdot t_{rise}} \right)$$

where:

$H_{peak}$  is the peak magnetic field value of the measured pulse in A/m.

$t_{rise}$  is the rise-time 10%-90% of the measured pulse in s.

$A_{eq,s}$  is the total equivalent area of the sensor in  $m^2$ .

$V_{max, transmitter}$  is the maximum allowed voltage at the input of the transmitter in V.

$\mu_0$  is the permeability of free space ( $1.256 \times 10^{-6}$  H/m).